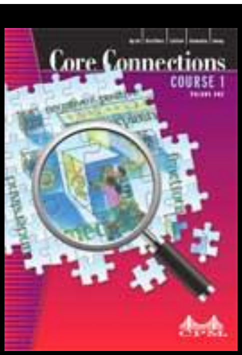


Chapter 4

Variables & Ratios

Extra Practice Packet For The Chapter 4 Test

Variable Expressions

Using Variables To Generalize

Addition Of Mixed Numbers

Subtraction Of Mixed Numbers

Addition & Subtraction Of Mixed Numbers Word Problems

Substitution & Evaluation Of Expressions

Scaling Figures & Scale Factor

Long Division

Multiplying Decimals

Dividing Decimals

VARIABLE EXPRESSIONS 4.1.1 – 4.1.3

A variable is a symbol used to represent one or more numbers. It is common to use letters of the alphabet for variables. The value of a variable used several times in one expression must be the same.

Example 1

If the unknown distance of Cecil's hop is represented by the variable h, write an expression for:

a. Three equal hops $\Rightarrow$ $h+h+h$ or $3h$

b. Five equal hops $\Rightarrow$ $h+h+h+h+h$ or $5h$

c. Two equal hops and walking 3 feet $\Rightarrow$ $h+h+3$ or $2h+3$

Example 2

If the unknown cost of a banana is b, and the unknown cost of an apple is a, write an expression for the cost of:

a. Three bananas and two apples $\Rightarrow$ $b+b+b+a+a$ or $3b+2a$

b. One banana and 3 apples $\Rightarrow$ $b+a+a+a$ or $b+3a$

c. One banana, one apple, a \$2 item, and a \$3 item $\Rightarrow$ $b+a+2+3$ or $b+a+5$

Problems

If the unknown distance of Cecil's jump is represented by J, write an expression for:

1. Three jumps
2. Six jumps
3. Four jumps and walking 5 feet
4. Walking 3 feet, two jumps, walking 2 feet

If the unknown distance of Cecil's jump is represented by J, and the unknown distance of Cecil's hop is represented by H, write an expression for:

5. Two jumps and two hops
6. One jump, three hops, and two jumps
7. One jump, three hops and walking 7 feet
8. Walking 6 feet, three hops, and two jumps

If the unknown cost of a taco is T, and the unknown cost of a carton of milk is M, write an expression for the cost of:

9. Three tacos and two milks
10. One taco and four milks
11. One taco, one milk and two tacos, one milk
12. Two tacos, one milk, and a $2 item

Answers

1. $J + J + J = 3J$
2. $J + J + J + J + J + J = 6J$
3. $J + J + J + J + 5 = 4J + 5$
4. $3 + J + J + 2 = 2J + 5$
5. $J + J + H + H = 2J + 2H$
6. $J + H + H + H + J + J = 3J + 3H$
7. $J + H + H + H + 7 = J + 3H + 7$
8. $6 + H + H + H + J + J = 3H + 2J + 6$
9. $3T + 2M$
10. $T + 4M$
11. $T + M + 2T + M = 3T + 2M$
12. $2T + M + 2$

USING VARIABLES TO GENERALIZE 4.1.1 – 4.1.3

Previously, students extended patterns and predicted subsequent figures. Now students use their knowledge of variables to generalize the pattern they observe. For additional information, see the Math Notes box in Lesson 4.2.1 of the *Core Connections, Course 1* text.

Example 1

Examine the dot pattern below. Draw the next figure, state the number of dots in Figure 15, and give a variable expression for the number of dots in figure "n."

Figure 1

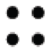


Figure 2

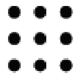


Figure 3

The next figure is:

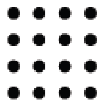


Figure 4

Each figure is a square with a side length that is the same as the figure number, so Figure 15 has $15 \cdot 15 = 225$ dots and figure n would have $n \cdot n = n^2$ dots.

Example 2

Examine the dot pattern below. Draw the next figure, state the number of dots in Figure 15, and give a variable expression for the number of dots in figure "n."

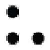


Figure 1

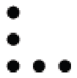


Figure 2

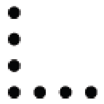


Figure 3

The next figure is:

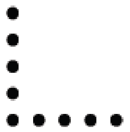


Figure 4

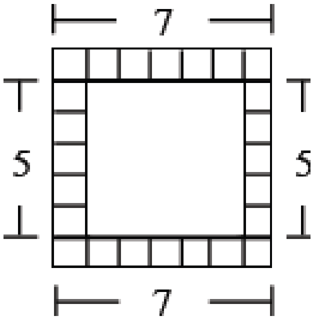
Each figure is an L shape with the number of dots one more than twice the figure number of dots so figure 15 has 15 + 15 + 1 = 31 dots. Figure n would have $2n+1$ dots. Another way to see the pattern is as the figure number plus "the figure number plus 1." This pattern then gives 15 + 16 = 31 for Figure 15 and $n+(n+1)$ for Figure n. Of course, $n+(n+1)=2n+1$.

USING VARIABLES TO GENERALIZE

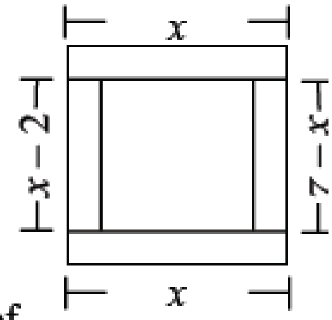
Variables are letters or symbols used to represent one or more numbers. They are often used to generalize patterns from a few specific numbers to include all possible numbers.

For example, if a square is surrounded by smaller square tiles each measuring one centimeter on a side, how many tiles are needed? It helps to look at a specific of size square first.

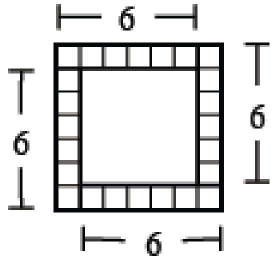
The outside square at right has side length 7. One way to see the total number of tiles needed for the frame is to consider that it needs 7 tiles for each of the top and bottom sides and $7 - 2 = 5$ tiles for the left and right sides. This is shown in the first diagram at right. The total number of tiles needed for the frame can be counted as $7 + 7 + 5 + 5 = 24$.

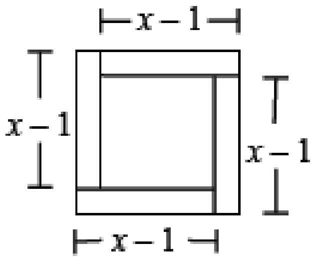


Square frames with different side lengths will follow the same pattern. You can generalize by writing an expression for any side length, denoted by x. The second diagram at right shows that the top and bottom each contain x tiles. The right and left sides each contain $x - 2$ tiles. You could write the total number of tiles as either $x + x + (x - 2) + (x - 2)$, $2x + 2(x - 2)$, or even as $4x - 4$.



Shown at right are two additional square-frame diagrams. The diagram on the left shows another way to count the number of tiles in a frame. The diagram on the right shows the algebraic expression associated with it. Notice that the expression resulting from this counting method could be written $(x - 1) + (x - 1) + (x - 1) + (x - 1)$, or $4(x - 1)$.





Problems

Examine each dot pattern below. Draw the next figure, tell the number of dots in figure 15, and give a variable expression for the number of dots in figure “n.”

1.

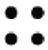


Figure 1

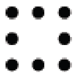


Figure 2

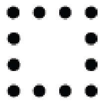


Figure 3

2.

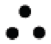


Figure 1

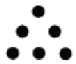


Figure 2

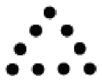


Figure 3

3.

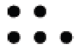


Figure 1

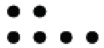


Figure 2

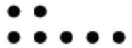


Figure 3

4.


Figure 1

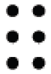


Figure 2

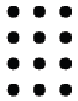


Figure 3

5.


Figure 1


Figure 2

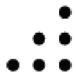


Figure 3

Answers Note: In each answer, n represents the figure number.

1.

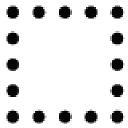


Figure 4

60 dots

$4 \cdot n = 4n$ dots

2.

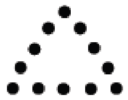


Figure 4

45 dots

$3 \cdot n = 3n$ dots

3.


Figure 4

19 dots

$n + 4$ dots

The base of each figure is 4 dots, plus the number of dots that matches the figure number.

4.

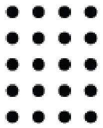


Figure 4

240 dots

$n \cdot (n+1)$ dots

5.

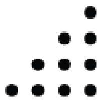


Figure 4

120 dots

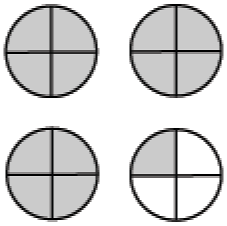
$\frac{n \cdot (n+1)}{2}$ dots

The figures in this pattern are half of the figures in problem 4.

MIXED NUMBERS AND FRACTIONS GREATER THAN ONE

The number $3\frac{1}{4}$ is called a **mixed number** because it is composed of a whole number, 3, and a fraction, $\frac{1}{4}$.

The number $\frac{13}{4}$ is called a **fraction greater than one** because the numerator, which represents the number of equal pieces, is larger than the denominator, which represents the number of pieces in one whole, so its value is greater than one. (Sometimes such fractions are called "improper fractions," but this is just a historical term. There is nothing actually wrong with the fractions.)



As you can see in the diagram at right, the fraction $\frac{13}{4}$ can be rewritten as $\frac{4}{4}+\frac{4}{4}+\frac{4}{4}+\frac{1}{4}$, which shows that it is equal in value to $3\frac{1}{4}$.

Your choice: Depending on which arithmetic operations you need to perform, you will choose whether to write your number as a mixed number or as a fraction greater than one.

ADDING AND SUBTRACTING MIXED NUMBERS

To **add or subtract mixed numbers**, you can either add or subtract their parts, or you can change the mixed numbers into fractions greater than one.

To add or subtract mixed numbers by adding or subtracting their parts, add or subtract the whole-number parts and the fraction parts separately. Adjust if the fraction in the answer would be greater than one or less than zero. For example, the sum of $3\frac{4}{5}+1\frac{2}{3}$ is calculated at right above.

$$\begin{aligned} 3\tfrac{4}{5} &= 3+\tfrac{4}{5}\cdot\boxed{\tfrac{3}{3}} = 3\tfrac{12}{15} \\ +1\tfrac{2}{3} &= 1+\tfrac{2}{3}\cdot\boxed{\tfrac{5}{5}} = \underline{+1\tfrac{10}{15}} \\ & \qquad\qquad 4\tfrac{22}{15} = 5\tfrac{7}{15} \end{aligned}$$

It is also possible to add or subtract mixed numbers by first changing them into fractions greater than one. Then add or subtract in the same way you would if they were fractions between 0 and 1. For example, the sum of $2\frac{1}{6}+1\frac{4}{5}$ is calculated at right.

$$\begin{aligned} 2\tfrac{1}{6}+1\tfrac{4}{5} &= \tfrac{13}{6}+\tfrac{9}{5} \\ &= \tfrac{13}{6}\cdot\boxed{\tfrac{5}{5}}+\tfrac{9}{5}\cdot\boxed{\tfrac{6}{6}} \\ &= \tfrac{65}{30}+\tfrac{54}{30} \\ &= \tfrac{119}{30} \\ &= 3\tfrac{29}{30} \end{aligned}$$

OPERATIONS WITH FRACTIONS 4.1.3 Math Note

ADDITION AND SUBTRACTION OF MIXED NUMBERS

To subtract mixed numbers, change the mixed numbers into fractions greater than one, find a common denominator, then subtract. Other strategies are also possible. For additional information, see the Math Notes box in Lesson 4.1.3 of the *Core Connections, Course 1* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 4 materials.

Example

Find the difference: $3\frac{1}{5} - 1\frac{2}{3}$

$$\begin{aligned} 3\tfrac{1}{5} &= \tfrac{16}{5} \cdot \boxed{\tfrac{3}{3}} = \tfrac{48}{15} \\ -1\tfrac{2}{3} &= \tfrac{5}{3} \cdot \boxed{\tfrac{5}{5}} = -\tfrac{25}{15} \\ &= \tfrac{23}{15} = 1\tfrac{8}{15} \end{aligned}$$

Problems

Find each difference.

1. $2\frac{1}{2} - 1\frac{3}{4}$
2. $4\frac{1}{3} - 3\frac{5}{6}$
3. $1\frac{1}{6} - \frac{3}{4}$
4. $5\frac{2}{5} - 3\frac{2}{3}$
5. $7 - 1\frac{2}{3}$
6. $5\frac{3}{8} - 2\frac{2}{3}$

Answers

1. $\frac{5}{2} - \frac{7}{4} \Rightarrow \frac{10}{4} - \frac{7}{4} \Rightarrow \frac{3}{4}$
2. $\frac{13}{3} - \frac{23}{6} \Rightarrow \frac{26}{6} - \frac{23}{6} \Rightarrow \frac{3}{6}$ or $\frac{1}{2}$
3. $\frac{7}{6} - \frac{3}{4} \Rightarrow \frac{14}{12} - \frac{9}{12} \Rightarrow \frac{5}{12}$
4. $\frac{27}{5} - \frac{11}{3} \Rightarrow \frac{81}{15} - \frac{55}{15} \Rightarrow \frac{26}{15}$ or $1\frac{11}{15}$
5. $\frac{7}{1} - \frac{5}{3} \Rightarrow \frac{21}{3} - \frac{5}{3} \Rightarrow \frac{16}{3}$ or $5\frac{1}{3}$
6. $\frac{43}{8} - \frac{8}{3} \Rightarrow \frac{129}{24} - \frac{64}{24} \Rightarrow \frac{65}{24}$ or $2\frac{17}{24}$

To add mixed numbers, it is possible to change the mixed numbers into fractions greater than one, find a common denominator, then add. Many times it is more efficient to add the whole numbers, add the fractions after getting a common denominator, and then simplify the answer. For additional information, see the Math Notes box in Lesson 4.1.3 of the *Core Connections, Course 1* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 4 materials.

Example

Find the sum: $8\frac{3}{4} + 4\frac{2}{5}$

$$\begin{array}{rcl} 8\frac{3}{4} = 8 + \frac{3}{4}\cdot\frac{5}{5} & = & 8\frac{15}{20} \\ +4\frac{2}{5} = 4 + \frac{2}{5}\cdot\frac{4}{4} & = & +4\frac{8}{20} \\ \hline & & 12\frac{23}{20} = 13\frac{3}{20} \end{array}$$

Problems

Find each sum.

1. $5\frac{3}{4} + 3\frac{1}{6}$
2. $5\frac{2}{3} + 8\frac{3}{8}$
3. $4\frac{4}{9} + 5\frac{2}{3}$
4. $1\frac{2}{5} + 3\frac{5}{6}$
5. $4\frac{1}{2} + 5\frac{3}{8}$
6. $5\frac{4}{7} + 3\frac{2}{3}$

Answers

1. $8\frac{11}{12}$
2. $13\frac{25}{24} = 14\frac{1}{24}$
3. $9\frac{10}{9} = 10\frac{1}{9}$
4. $4\frac{37}{30} = 5\frac{7}{30}$
5. $9\frac{7}{8}$
6. $8\frac{26}{21} = 9\frac{5}{21}$

To add mixed numbers, it is possible to change the mixed numbers into fractions greater than one, find a common denominator, then add. Many times it is more efficient to add the whole numbers, add the fractions after getting a common denominator, and then simplify the answer. For additional information, see the Math Notes box in Lesson 4.1.3 of the *Core Connections, Course 1* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 4 materials.

Example

Find the sum: $8\frac{3}{4} + 4\frac{2}{5}$

$$\begin{array}{rl} 8\frac{3}{4} = 8 + \frac{3}{4} \cdot \boxed{\frac{5}{5}} = & 8\frac{15}{20} \\ +4\frac{2}{5} = 4 + \frac{2}{5} \cdot \boxed{\frac{4}{4}} & \underline{= +4\frac{8}{20}} \\ & 12\frac{23}{20} = 13\frac{3}{20} \end{array}$$

Problems

Find each sum.

1. $5\frac{3}{4} + 3\frac{1}{6}$
2. $5\frac{2}{3} + 8\frac{3}{8}$
3. $4\frac{4}{9} + 5\frac{2}{3}$
4. $1\frac{2}{5} + 3\frac{5}{6}$
5. $4\frac{1}{2} + 5\frac{3}{8}$
6. $5\frac{4}{7} + 3\frac{2}{3}$

Answers

1. $8\frac{11}{12}$
2. $13\frac{25}{24} = 14\frac{1}{24}$
3. $9\frac{10}{9} = 10\frac{1}{9}$
4. $4\frac{37}{30} = 5\frac{7}{30}$
5. $9\frac{7}{8}$
6. $8\frac{26}{21} = 9\frac{5}{21}$

Adding Mixed Fractions (B)

Find the value of each expression in lowest terms.

1. $3\frac{5}{8}+5\frac{5}{8}$

2. $4\frac{13}{20}+5\frac{3}{4}$

5. $1\frac{1}{4}+19\frac{1}{2}$

6. $1\frac{3}{5}+14\frac{4}{5}$

9. $5\frac{3}{4}+14\frac{1}{2}$

10. $1\frac{1}{2}+1\frac{1}{3}$

Adding Mixed Fractions (B) Answers

Find the value of each expression in lowest terms.

1. $3\frac{5}{8}+5\frac{5}{8}$
$=\frac{37}{4}=9\frac{1}{4}$

2. $4\frac{13}{20}+5\frac{3}{4}$
$=\frac{52}{5}=10\frac{2}{5}$

5. $1\frac{1}{4}+19\frac{1}{2}$
$=\frac{83}{4}=20\frac{3}{4}$

6. $1\frac{3}{5}+14\frac{4}{5}$
$=\frac{82}{5}=16\frac{2}{5}$

9. $5\frac{3}{4}+14\frac{1}{2}$
$=\frac{81}{4}=20\frac{1}{4}$

10. $1\frac{1}{2}+1\frac{1}{3}$
$=\frac{17}{6}=2\frac{5}{6}$

Subtracting Mixed Fractions (A)

Find the value of each expression in lowest terms.

1. $11\frac{1}{3} - 6\frac{11}{12}$

2. $8\frac{2}{5} - 7\frac{2}{11}$

5. $26\frac{1}{3} - 5\frac{19}{21}$

6. $7\frac{1}{2} - 4\frac{1}{4}$

9. $8\frac{2}{5} - 3\frac{37}{40}$

10. $3\frac{1}{3} - 2\frac{29}{39}$

Subtracting Mixed Fractions (A) Answers

Find the value of each expression in lowest terms.

1. $11\frac{1}{3} - 6\frac{11}{12}$
$= \frac{53}{12} = 4\frac{5}{12}$

2. $8\frac{2}{5} - 7\frac{2}{11}$
$= \frac{67}{55} = 1\frac{12}{55}$

5. $26\frac{1}{3} - 5\frac{19}{21}$
$= \frac{143}{7} = 20\frac{3}{7}$

6. $7\frac{1}{2} - 4\frac{1}{4}$
$= \frac{13}{4} = 3\frac{1}{4}$

9. $8\frac{2}{5} - 3\frac{37}{40}$
$= \frac{179}{40} = 4\frac{19}{40}$

10. $3\frac{1}{3} - 2\frac{29}{39}$
$= \frac{23}{39}$

Adding & Subtracting Fractions

Name:

Determine the best answer for each question.

1) A restaurant had $5\frac{1}{12}$ ounces of soup at the start of the day. By the end of the day they had $3\frac{2}{5}$ ounces left. How many ounces of soup did they use during the day?

2) While exercising Carl jogged $3\frac{2}{12}$ kilometers and walked $9\frac{5}{8}$ kilometers. What is the total distance he traveled?

3) A king size chocolate bar was $6\frac{4}{8}$ inches long. The regular size bar was $4\frac{5}{8}$ inches long. What is the difference in length between the two bars?

4) Bobby drew a line that was $5\frac{8}{10}$ inches long. If he drew a second line that was $3\frac{5}{10}$ inches longer, what is the length of the second line?

5) A chef had $6\frac{6}{8}$ pounds of carrots. If he later used $2\frac{5}{12}$ pounds in a recipe, how many pounds of carrots does he have left?

6) An architect built a road $8\frac{3}{8}$ miles long. The next road he built was $6\frac{4}{8}$ miles long. What is the combined length of the two roads?

7) Isaac jogged $5\frac{9}{12}$ kilometers on Monday and $3\frac{6}{10}$ kilometers on Tuesday. What is the difference between these two distances?

8) Over the weekend Janet spent $9\frac{1}{5}$ hours total studying. If she spent $8\frac{3}{10}$ hours studying on Saturday, how long did she study on Sunday?

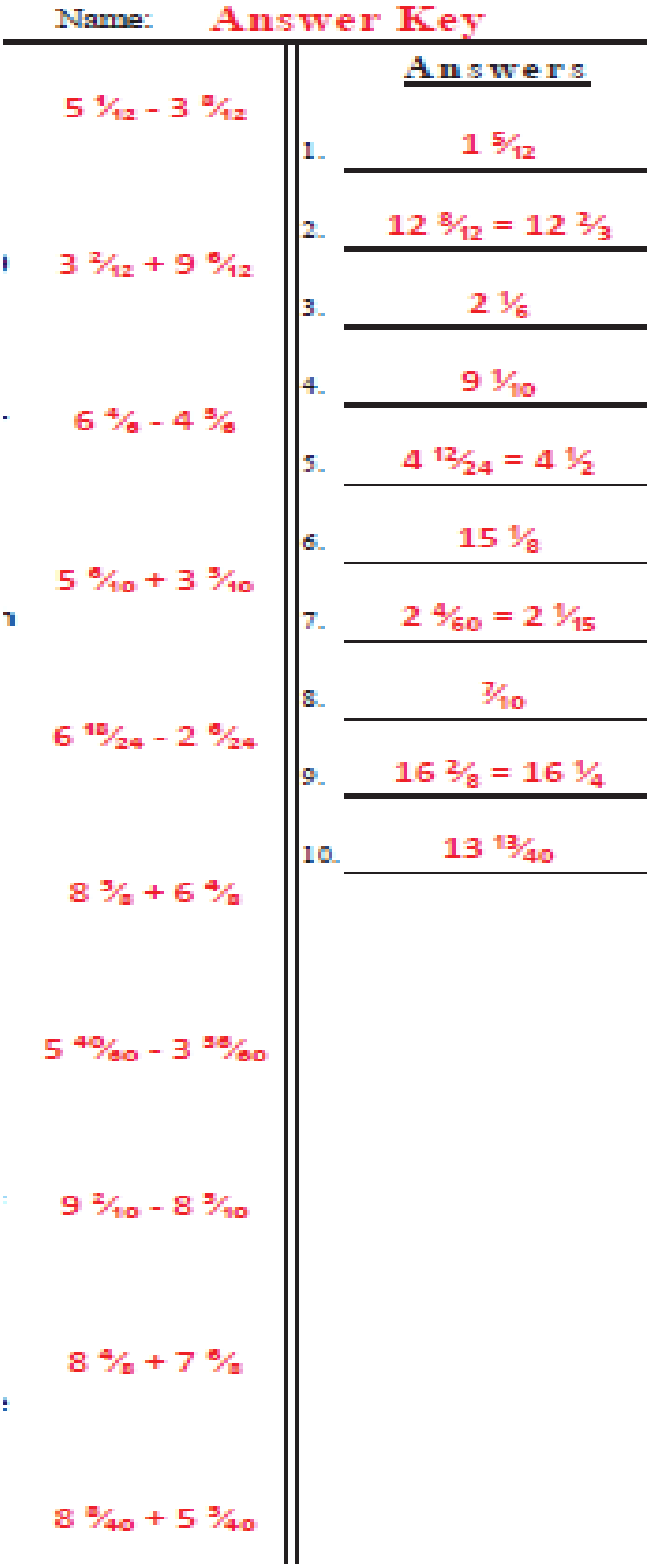
9) At the beach, Victor built a sandcastle that was $8\frac{4}{8}$ feet high. If he added a flag that was $7\frac{6}{8}$ feet high, what is the total height of his creation?

10) A small box of nails was $8\frac{1}{5}$ inches tall. If the large box of nails was $5\frac{1}{8}$ inches taller, how tall is the large box of nails?

Answers

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______

Fraction word problems answer key : >>>>>>>>>>>>>>>>



Name: **Answer Key**

$5\frac{1}{12} - 3\frac{8}{12}$

$3\frac{2}{12} + 9\frac{6}{12}$

$6\frac{4}{6} - 4\frac{3}{6}$

$5\frac{6}{10} + 3\frac{5}{10}$

$6\frac{18}{24} - 2\frac{6}{24}$

$8\frac{5}{8} + 6\frac{4}{8}$

$5\frac{40}{60} - 3\frac{36}{60}$

$9\frac{2}{10} - 8\frac{5}{10}$

$8\frac{4}{8} + 7\frac{6}{8}$

$8\frac{8}{40} + 5\frac{5}{40}$

Answers

1. $1\frac{5}{12}$
2. $12\frac{8}{12} = 12\frac{2}{3}$
3. $2\frac{1}{6}$
4. $9\frac{1}{10}$
5. $4\frac{12}{24} = 4\frac{1}{2}$
6. $15\frac{1}{8}$
7. $2\frac{4}{60} = 2\frac{1}{15}$
8. $\frac{7}{10}$
9. $16\frac{2}{8} = 16\frac{1}{4}$
10. $13\frac{13}{40}$

EVALUATING ALGEBRAIC EXPRESSIONS

An **algebraic expression**, also known as a *variable expression*, is a combination of numbers and variables, connected by mathematical operations. For example, $4x$, $3(x - 5)$, and $4x - 3y + 7$ are algebraic expressions.

Addition and subtraction separate expressions into parts called **terms**. For example the expression above, $4x - 3y + 7$, has three terms: $4x$, $-3y$, and 7.

A more complex expression is $2x + 3(5 - 2x) + 8$. It also has three terms: $2x$, $3(5 - 2x)$, and 8. But the term $3(5 - 2x)$ has another expression, $5 - 2x$, inside the parentheses. The terms of this inner expression are 5 and $-2x$.

To **evaluate** an algebraic expression for particular values of variables, replace the variables in the expression with their known numerical values and simplify. Replacing variables with their known values is called **substitution**. An example is provided below.

> Evaluate $4x - 3y + 7$ for $x = 2$ and $y = 1$.
>
> Replace x and y with their known values of 2 and 1, respectively, and simplify.

$$\begin{gathered} 4(2) - 3(1) + 7 \\ 8 - 3 + 7 \\ 12 \end{gathered}$$

SUBSTITUTION AND EVALUATION OF EXPRESSIONS 4.1.3

Substitution is replacing one symbol with an equivalent symbol (a number, a variable, or an expression). One application of the substitution property is replacing a variable name with a number in an expression or equation. A variable is a letter used to represent one or more numbers (or other algebraic expression). The numbers are the values of the variable. A variable expression has numbers and variables with arithmetic operations performed on it.

In general, if $a = b$, then a may replace b and b may replace a.

After numerical substitutions have been made, following the order of operations and doing the arithmetic will correctly evaluate the expression.

For additional information, see the Math Notes box in Lesson 4.2.2 of the *Core Connections, Course 1* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 8A materials.

Examples

Evaluate each variable expression for $x = 3$.

a. $5x \Rightarrow 5(3) \Rightarrow 15$

b. $x + 10 \Rightarrow (3) + 10 \Rightarrow 13$

c. $\frac{18}{x} \Rightarrow \frac{18}{3} \Rightarrow 6$

d. $\frac{x}{3} \Rightarrow \frac{3}{3} \Rightarrow 1$

e. $3x - 5 \Rightarrow 3(3) - 5 \Rightarrow 9 - 5 \Rightarrow 4$

f. $5x + 3x \Rightarrow 5(3) + 3(3) \Rightarrow 15 + 9 \Rightarrow 24$

Problems

Evaluate each of the variable expressions below for $x = -4$ and $y = 3$. Be sure to follow the Order of Operations as you simplify each expression.

1. $x + 4$
2. $x - 1$
3. $x + y + 3$
4. $y - 3 + x$
5. $x^2 - 5$
6. $-x^2 + 5$
7. $x^2 + 4x - 3$
8. $-3x^2 + 2x$
9. $x + 3 + 2y$
10. $y^2 + 3x - 2$
11. $x^2 + y^2 + 3^2$
12. $2^2 + y^2 - 2x^2$

Evaluate the expressions below using the values of the variables in each problem. These problems ask you to evaluate each expression twice, once with each of the values.

13. $3x^2 - 2x + 5$ for $x = -3$ and $x = 4$

14. $2x^2 - 3x + 6$ for $x = -2$ and $x = 5$

15. $-3x^2 + 7$ for $x = -3$ and $x = 2$

16. $-2x^2 + 5$ for $x = -4$ and $x = 5$

Evaluate the variable expressions for $x = -4$ and $y = 3$.

17. $x(x + 3x)$

18. $2(x + 2x)$

19. $2(x + y) + 4\left(\frac{y+2}{x}\right)$

20. $3\left(y^2 + 2\left(\frac{x+7}{2}\right)\right)$

21. $2y\left(x + x^2 - 2y\right)$

22. $(3x + y)(2x + 4y)$

Answers

1. 0 2. –5 3. 2 4. –4 5. 11

6. –11 7. –3 8. –56 9. 5 10. –5

11. 34 12. –19 13. 38; 45 14. 20; 41 15. –20; –5

16. –27; –45 17. 36 18. –18 19. 20 20. 36

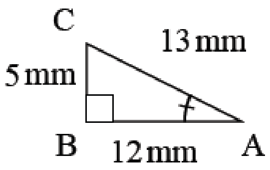
21. –16 22. –50

SCALING FIGURES AND SCALE FACTOR 4.2.1 – 4.2.3

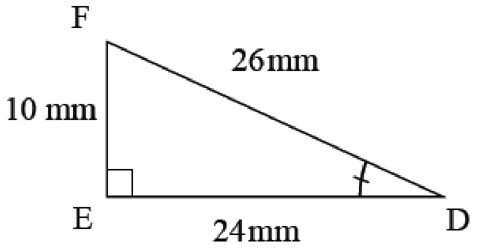
Geometric figures can be reduced or enlarged. When this change happens, every length of the figure is reduced or enlarged equally (proportionally), and the measures of the corresponding angles stay the same.

The ratio of any two corresponding sides of the original and new figure is called a scale factor. The scale factor may be written as a percent or a fraction. It is common to write new figure measurements over their original figure measurements in a scale ratio, that is, $\frac{\text{NEW}}{\text{ORIGINAL}}$.

Example 1 using a 200% enlargement



original triangle



new triangle

Side length ratios:

$$\frac{DE}{AB} = \frac{24}{12} = \frac{2}{1}$$

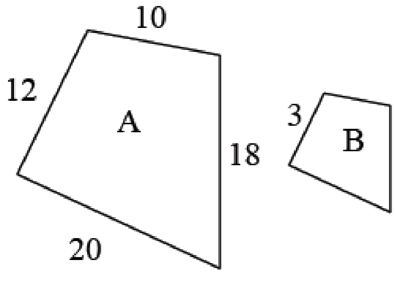
$$\frac{FD}{CA} = \frac{26}{13} = \frac{2}{1}$$

$$\frac{FE}{CB} = \frac{10}{5} = \frac{2}{1}$$

The scale factor for length is 2 to 1.

Example 2

Figures A and B at right are similar. Assuming that Figure A is the original figure, find the scale factor and find the lengths of the missing sides of Figure B.

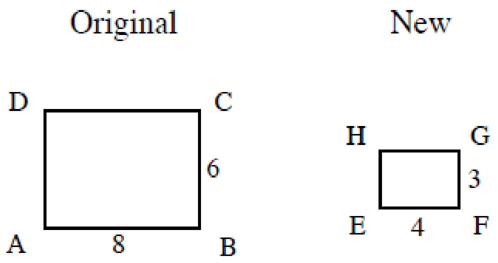


The scale factor is $\frac{3}{12} = \frac{1}{4}$. The lengths of the missing sides of Figure B are: $\frac{1}{4}(10) = 2.5$, $\frac{1}{4}(18) = 4.5$, and $\frac{1}{4}(20) = 5$.

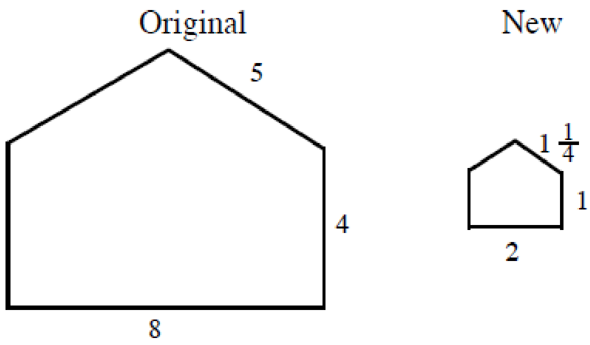
Problems

Determine the scale factor for each pair of similar figures in problems 1 through 4.

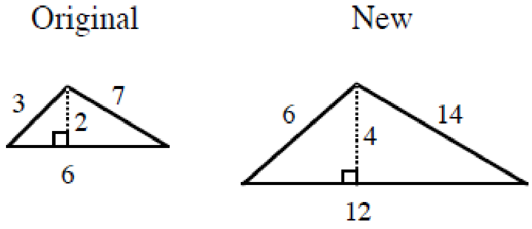
1.



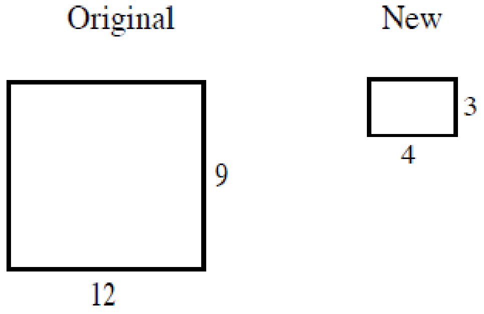
2.



3.



4.



5. A triangle has sides 5, 12, and 13. The triangle was enlarged by a scale factor of 300%.
   a. What are the lengths of the sides of the new triangle?
   b. What is the ratio of the perimeter of the new triangle to the perimeter of the original triangle?

6. A rectangle has a length of 60 cm and a width of 40 cm. The rectangle was reduced by a scale factor of 25%.
   a. What are the dimensions of the new rectangle?
   b. What is the ratio of the perimeter of the new rectangle to the perimeter of the original rectangle?

Answers

1. $\frac{4}{8} = \frac{1}{2}$
2. $\frac{2}{8} = \frac{1}{4}$
3. $\frac{2}{1}$
4. $\frac{1}{3}$
5. a. 15, 36, 39 b. $\frac{3}{1}$
6. a. 15 cm and 10 cm b. $\frac{1}{4}$

RATIOS

A **ratio** is a comparison of two numbers, often written as a quotient; that is, the first number is divided by the second number (but not zero). A ratio can be written in words, in fraction form, or with colon notation. Most often, in this class, you will either write ratios in the form of fractions or state the ratios in words.

For example, if there are 38 students in a school band and 16 of them are boys, you can write the ratio of the number of boys to the number of girls as:

16 boys to 22 girls $\qquad \dfrac{\text{16 boys}}{\text{22 girls}} \qquad$ 16 boys : 22 girls

RATIOS

A ratio is a comparison of two quantities by division. It can be written in several ways:

$\frac{\text{65 miles}}{\text{1 hour}}$, 65 miles: 1 hour, or 65 miles to 1 hour

For additional information see the Math Notes boxes in Lesson 4.2.4 of the *Core Connections, Course 1* text or Lesson 5.1.1 of the *Core Connections, Course 2* text.

Example

A bag contains the following marbles: 7 clear, 8 red and 5 blue. The following ratios may be stated:

a. Ratio of blue to total number of marbles $\Rightarrow \frac{5}{20} = \frac{1}{4}$.

b. Ratio of red to clear $\Rightarrow \frac{8}{7}$.

c. Ratio of red to blue $\Rightarrow \frac{8}{5}$.

d. Ratio of blue to red $\Rightarrow \frac{5}{8}$.

Problems

1. Molly's favorite juice drink is made by mixing 3 cups of apple juice, 5 cups of cranberry juice, and 2 cups of ginger ale. State the following ratios:

   a. Ratio of cranberry juice to apple juice.

   b. Ratio of ginger ale to apple juice.

   c. Ratio of ginger ale to finished juice drink (the mixture).

2. A 40-passenger bus is carrying 20 girls, 16 boys, and 2 teachers on a field trip to the state capital. State the following ratios:

   a. Ratio of girls to boys.

   b. Ratio of boys to girls.

   c. Ratio of teachers to students.

   d. Ratio of teachers to passengers.

3. It is important for Molly (from problem one) to keep the ratios the same when she mixes larger or smaller amounts of the drink. Otherwise, the drink does not taste right. If she needs a total of 30 cups of juice drink, how many cups of each liquid should be used?

4. If Molly (from problem one) needs 25 cups of juice drink, how many cups of each liquid should be used? Remember that the ratios must stay the same.

Answers

1. a. $\frac{5}{3}$ b. $\frac{2}{3}$ c. $\frac{2}{10} = \frac{1}{5}$

2. a. $\frac{20}{16} = \frac{5}{4}$ b. $\frac{16}{20} = \frac{4}{5}$ c. $\frac{2}{36}$ d. $\frac{2}{38}$

3. 9 c. apple, 15 c. cranberry, 6 c. ginger ale

4. $7\frac{1}{2}$ c. apple, $12\frac{1}{2}$ c. cranberry, 5 c. ginger ale

SCALING FIGURES AND SCALE FACTOR

Geometric figures can be reduced or enlarged. When this change happens, every length of the figure is reduced or enlarged equally (proportionally), and the measures of the corresponding angles stay the same.

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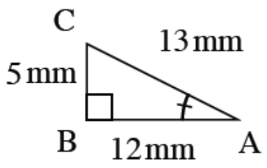
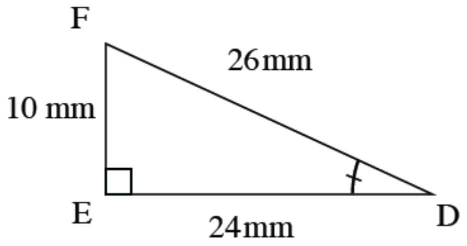
For additional information, see the Math Notes boxes in Lesson 4.1.2 of the *Core Connections, Course 2* text or Lesson 6.2.6 of the *Core Connections, Course 3* text.

Example 1 using a 200% enlargement



original triangle



new triangle

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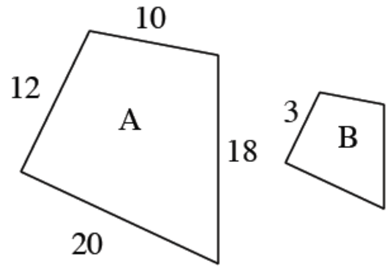


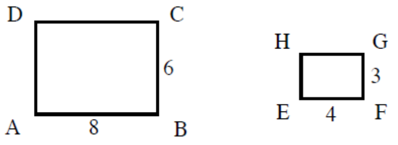
The scale factor is $\frac{3}{12} = \frac{1}{4}$. The lengths of the missing sides of Figure B are: $\frac{1}{4}(10) = 2.5$, $\frac{1}{4}(18) = 4.5$, and $\frac{1}{4}(20) = 5$.

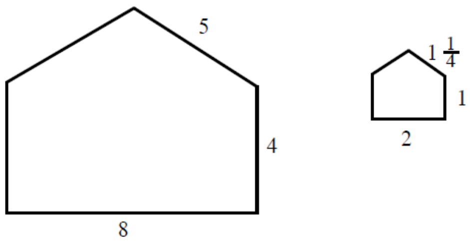
Problems

Determine the scale factor for each pair of similar figures in problems 1 through 4.

1. Original New

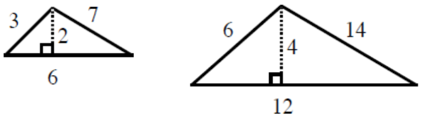
2. Original New





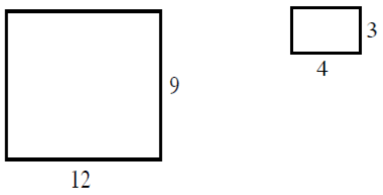
3\.

Original New



4\.

Original New



5\. A triangle has sides 5, 12, and 13. The triangle was enlarged by a scale factor of 300%.

   a. What are the lengths of the sides of the new triangle?

   b. What is the ratio of the perimeter of the new triangle to the perimeter of the original triangle?

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   a. What are the dimensions of the new rectangle?

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Answers

1\. $\frac{4}{8}=\frac{1}{2}$

2\. $\frac{2}{8}=\frac{1}{4}$

3\. $\frac{2}{1}$

4\. $\frac{1}{3}$

5\. a. 15, 36, 39 b. $\frac{3}{1}$

6\. a. 15 cm and 10 cm b. $\frac{1}{4}$

MATH NOTES

DIVIDING

When using long division to divide one number by another, it is important to be sure that you know the place value of each digit in your result.

In the example of dividing 225 by 6 at right, people often begin by saying, *"6 goes into 22 three times."* If they were paying attention to place value, they would instead say *"6 goes into 220 thirty-something times."* The 3 of the quotient is written in the tens place to indicate that 6 goes into 225 at least 30 times, but less than 40. The 3 represents 3 tens.

$$\begin{array}{r} 37 \\ 6\overline{)225} \\ -180 \\ \hline 45 \\ -42 \\ \hline 3 \end{array}$$

It may seem like the divisor is then multiplied by the 3, and the product, 18, is placed below a 22. However, you are really multiplying 30 by 6 and the product is 180, which is placed below 225. You would then subtract, getting what looks like 4. But then you would "bring down" the 5, to get 45. Notice that if you subtract 180 from 225, as in the top example at right, you get 45 directly. You then repeat the same process. In the past, you may have stopped at this point and written that the quotient is 37 with a remainder of 3.

$$\begin{array}{r} 37.5 \\ 6\overline{)225.0} \\ -180 \\ \hline 45 \\ -42 \\ \hline 30 \\ 30 \\ \hline 0 \end{array}$$

The same method works for dividing decimals. The bottom example at right is essentially the same as the top one, except that it shows what happens if you keep dividing past the decimal point, while still keeping place value in mind.

Long Division (4 ÷ 2) Name:

Use division to find the quotient for the following problems.

1) $64\overline{)4229}$

2) $27\overline{)2108}$

3) $90\overline{)6888}$

4) $95\overline{)5074}$

5) $63\overline{)6272}$

6) $35\overline{)3398}$

7) $32\overline{)9470}$

8) $47\overline{)5991}$

9) $32\overline{)5116}$

10) $94\overline{)5803}$

11) $17\overline{)2025}$

12) $63\overline{)9377}$

Answers

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______
11. ______
12. ______

Long Division (4 ÷ 2) Name: Answer Key

Use division to find the quotient for the following problems.

1)
0 0 6 6 r5
64 | 4 2 2 9
0 0
4 2 2
3 8 4
3 8 9
3 8 4
0 5

2)
0 0 7 8 r2
27 | 2 1 0 8
0 0
2 1 0
1 8 9
2 1 8
2 1 6
0 2

3)
0 0 7 6 r48
90 | 6 8 8 8
0 0
6 8 8
6 3 0
5 8 8
5 4 0
4 8

4)
0 0 5 3 r39
95 | 5 0 7 4
0 0
5 0 7
4 7 5
3 2 4
2 8 5
3 9

5)
0 0 9 9 r35
63 | 6 2 7 2
0 0
6 2 7
5 6 7
6 0 2
5 6 7
3 5

6)
0 0 9 7 r3
35 | 3 3 9 8
0 0
3 3 9
3 1 5
2 4 8
2 4 5
0 3

7)
0 2 9 5 r30
32 | 9 4 7 0
6 4
3 0 7
2 8 8
1 9 0
1 6 0
3 0

8)
0 1 2 7 r22
47 | 5 9 9 1
4 7
1 2 9
0 9 4
3 5 1
3 2 9
2 2

9)
0 1 5 9 r28
32 | 5 1 1 6
3 2
1 9 1
1 6 0
3 1 6
2 8 8
2 8

10)
0 0 6 1 r69
94 | 5 8 0 3
0 0
5 8 0
5 6 4
1 6 3
0 9 4
6 9

11)
0 1 1 9 r2
17 | 2 0 2 5
1 7
0 3 2
0 1 7
1 5 5
1 5 3
0 2

12)
0 1 4 8 r53
63 | 9 3 7 7
6 3
3 0 7
2 5 2
5 5 7
5 0 4
5 3

Answers

1. 66 r5
2. 78 r2
3. 76 r48
4. 53 r39
5. 99 r35
6. 97 r3
7. 295 r30
8. 127 r22
9. 159 r28
10. 61 r69
11. 119 r2
12. 148 r53

4) 0.62 × 9

5) 0.202 × 57

6) 0.145 × 4

7) 0.5 × 1

8) 0.05 × 22

9) 0.214 × 835

[illegible]

1) 6.6 | 5 4 8.0

2) .4 4 | 6 9.3 7

3) .1 2 | 7 0 4.1

Dividing Decimals

Name: **Answer Key**

Find the quotient to the following problems. Round your answer to the nearest whole number.

1)
0 0 8 3.0
6.6 | 5 4 8.0 0
0
5 4
0 0
5 4 8
5 2 8
2 0 0
1 9 8
2 0
0 0
2 0

2)
0 1 5 7.6
.4 4 | 6 9.3 7 0
0
6 9
4 4
2 5 3
2 2 0
3 3 7
3 0 8
2 9 0
2 6 4
2 6

3)
0 5 8 6 7.5
.1 2 | 7 0 4.1 0 0
0
7 0
6 0
1 0 4
9 6
0 8 1
7 2
9 0
8 4
6 6 0
6 0
0 0

4)
0 6 9 6 1.8
.1 1 | 7 6 5.8 0 0
0
7 6
6 6
1 0 5
9 9
0 6 8
6 6
2 0
1 1
9 9 0
8 8
0 2

5)
0 0 1 8.9
.7 2 | 1 3.6 5 0
0
1 3
0 0
1 3 6
7 2
6 4 5
5 7 6
6 9 0
6 4 8
4 2

6)
0 2 2.0
3.8 | 8 3.9 2
0
8 3
7 6
7 9
7 6
0 3 2
0 0
3 2

Answers

1. 83
2. 158
3. 5868
4. 6962
5. 19
6. 22